Int. J. Heat Mass Transfer Vol. 33, No. 9, pp. 2045–2049, 1990
Printed in Great Britain

0017-9310/90 $3.00+0.00
Pergamon Press plc

Influence of the interphase mass transfer on the rate of mass transfer—2. The system 'gas–liquid'

NIKOLAI L. VULCHANOV
Department of Mathematics, School for Mechanical and Electrical Engineering, 4 'H. Dimitar' Str., 5300 Gabrovo, Bulgaria

and

CHRISTO B. BOYADJIEV
Institute for Chemical Engineering, 'Akad. G. Bonchev' Str., Block 103, 1113 Sofia, Bulgaria

(*Received* 7 *September* 1988)

Abstract—Numerical results are reported for the influence of the direction of the interphase mass transfer on the rate of mass transfer in 'gas–liquid' systems. It is shown that this influence is significant when the diffusive resistance of the liquid phase is negligible. When the diffusive resistances in both phases are of the same order of magnitude the effect is weak. In all cases the intensive mass transfer from the gas to the liquid results in an increased rate of mass transfer compared to the results of the linear theory. The change of direction of mass transfer has the reverse effect.

1. INTRODUCTION

RECENTLY it was shown [1–4] that in the presence of intensive mass transfer the kinetics of mass transfer exhibits effects which cannot be described by the linear theory of diffusion in boundary layer approximation. A non-linear theory of mass transfer was therefore developed and applied to study the systems 'solid–gas (liquid)' [1, 2] and 'gas–liquid' [3, 4]. It was also known that the direction of the intensive mass transfer influences the rate of mass transfer. The purpose of this brief is to report results from the numerical study of the influence of the direction of the interphase mass transfer and its relation to the non-linear effects in gas–liquid systems.

2. THE MATHEMATICAL MODEL

The rate of mass transfer in gas–liquid systems [3–5] can be expressed by the rates of mass transfer in the gas and the liquid in the form

$$J = Mk_1(c_{10}-\mathrm{X}c_{20}) = L^{-1}\int_0^L I_1\,\mathrm{d}x$$

$$= Mk_2(c_{10}/\mathrm{X}-c_{20}) = L^{-1}\int_0^L I_2\,\mathrm{d}x \qquad (1)$$

where

$$I_1 = -\frac{MD_1\rho_1^*}{\rho_{10}^*}\left(\frac{\partial c_1}{\partial y}\right)_{y=0} \qquad (2a)$$

$$I_2 = -MD_2\left(\frac{\partial c_2}{\partial y}\right)_{y=0}. \qquad (2b)$$

From equations (1) and (2) one can readily express the Sherwood numbers for both phases as

$$Sh_1 = \frac{k_1L}{D_1} = -\frac{\rho_1^*}{\rho_{10}^*}\frac{1}{c_{10}-\mathrm{X}c_{20}}\int_0^L\left(\frac{\partial c_1}{\partial y}\right)_{y=0}\mathrm{d}x \qquad (3a)$$

$$Sh_2 = \frac{k_2L}{D_2} = -\frac{1}{c_{10}/\mathrm{X}-c_{20}}\int_0^L\left(\frac{\partial c_2}{\partial y}\right)_{y=0}\mathrm{d}x. \qquad (3b)$$

Therefore, to determine the overall rate of mass transfer J, one has to find the concentration distributions c_1 and c_2 in both phases. This problem will be solved in the boundary layer approximation as in refs. [1–3]. In the coordinate system described in these papers the momentum and the convective diffusion equations have the form

$$u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y} = \nu_1\frac{\partial^2 u_1}{\partial y^2}, \quad x>0,\ y>0 \qquad (4a)$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y} = 0, \quad x>0,\ y>0 \qquad (4b)$$

$$u_1\frac{\partial c_1}{\partial x}+v_1\frac{\partial c_1}{\partial y} = D_1\frac{\partial^2 c_1}{\partial y^2}, \quad x>0,\ y>0 \qquad (4c)$$

$$u_2\frac{\partial u_2}{\partial x}+v_2\frac{\partial u_2}{\partial y} = \nu_2\frac{\partial^2 u_2}{\partial y^2}, \quad x>0,\ y>0 \qquad (4d)$$

$$\frac{\partial u_2}{\partial x}+\frac{\partial v_2}{\partial y} = 0, \quad x>0,\ y>0 \qquad (4e)$$

$$u_2\frac{\partial c_2}{\partial x}+v_2\frac{\partial c_2}{\partial y} = D_2\frac{\partial^2 c_2}{\partial y^2}, \quad x>0,\ y>0. \qquad (4f)$$

The boundary conditions for the system of non-linear partial differential equations (4) are similar to

NOMENCLATURE

- c — molar concentration of the absorbed substance
- D — diffusivity
- I — local mass transfer rate at the phase boundary
- J — overall mass flux at the phase boundary
- L — length of the phase boundary
- k — mass transfer coefficient
- M — molecular mass of the absorbed substance
- u — local velocity component, parallel to the phase boundary
- v — local velocity component, normal to the phase boundary
- x — coordinate, parallel to the phase boundary
- y — coordinate, normal to the phase boundary.

Greek symbols

- μ — dynamic viscosity
- ν — kinematic viscosity
- ρ — density
- X — Henry's constant.

Subscripts

- ∞ — far from the phase boundary
- 0 — beginning of the phase boundary or fluid density in the absence of absorbed substance
- 1 — gas
- 2 — liquid
- k — 1 or 2.

Superscript

- $*$ — at the phase boundary or equilibrium value.

the ones described in refs. [1–3], but in this case the non-linear effects in the liquid phase are considered negligible following the results [4]. If this effect has to be taken into consideration, it can be shown that the above equations are not valid for the gas phase. Thus one has

$$u_1 = u_{10}; \quad x = 0, y > 0 \tag{5a}$$

$$u_2 = u_{20}; \quad x = 0, y < 0 \tag{5b}$$

$$c_1 = c_{10}; \quad x = 0, y > 0 \tag{5c}$$

$$c_2 = c_{20}; \quad x = 0, y < 0 \tag{5d}$$

$$u_1 = u_{10}; \quad x > 0, y \to \infty \tag{5e}$$

$$u_2 = u_{20}; \quad x > 0, y \to -\infty \tag{5f}$$

$$c_1 = c_{10}; \quad x > 0, y \to \infty \tag{5g}$$

$$c_2 = c_{20}; \quad x > 0, y \to -\infty \tag{5h}$$

$$u_1 = u_1; \quad x > 0, y = 0 \tag{5i}$$

$$\mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y}; \quad x > 0, y = 0 \tag{5j}$$

$$v_1 = -\frac{MD_1}{\rho_{10}^*} \frac{\partial c_1}{\partial y}; \quad x > 0, y = 0 \tag{5k}$$

$$v_2 = 0; \quad x > 0, y = 0 \tag{5l}$$

$$c_1 = \mathrm{X} c_2; \quad x > 0, y = 0 \tag{5m}$$

$$\frac{D_1 \rho_1^*}{\rho_{10}^*} \frac{\partial c_1}{\partial y} = D_2 \frac{\partial c_2}{\partial y}; \quad x > 0, y = 0. \tag{5n}$$

To solve the boundary value problem, equations (4) and (5), where the partial differential equations (4) are coupled by the boundary conditions, equation (5), the following similarity variables are introduced:

$$u_1 = 0.5 u_{10} \varepsilon_1 \Phi_1' \tag{6a}$$

$$v_1 = 0.5 \left(\frac{u_{10} v_1}{x} \right)^{0.5} (\eta_1 \Phi_1' - \Phi_1) \tag{6b}$$

$$c_1 = c_{10} - (c_{10} - \mathrm{X} c_{20}) \Psi_1 \tag{6c}$$

$$\Phi_1 = \Phi_1(\eta_1), \quad \Psi_1 = \Psi_1(\eta_1) \tag{6d}$$

$$\eta_1 = y \left(\frac{u_{10}}{4 D_1 x} \right)^{0.5}; \quad y > 0 \tag{6e}$$

$$u_2 = u_{20} \varepsilon_2 \Phi_2' \tag{6f}$$

$$v_2 = -\left(\frac{u_{20} v_2}{x} \right)^{0.5} (\eta_2 \Phi_2' - \Phi_2) \tag{6g}$$

$$c_2 = c_{20} + (c_{10}/\mathrm{X} - c_{20}) \Psi_2 \tag{6h}$$

$$\Phi_2 = \Phi_2(\eta_2), \quad \Psi_2 = \Psi_2(\eta_2) \tag{6i}$$

$$\eta_2 = -y \left(\frac{u_{20}}{4 D_2 x} \right)^{0.5}; \quad y < 0 \tag{6j}$$

$$\varepsilon_k = (Sc_k)^{0.5}, \quad Sc_k = v_k / D_k; \quad k = 1, 2. \tag{6k}$$

Substitution of equations (6) into equations (4) and (5) reduces the boundary value problem, equations (4) and (5), into a two-point boundary value problem for a system of ordinary differential equations, defined on a semiinfinite interval

$$\Phi_1''' + \varepsilon_1^{-1} \Phi_1 \Phi_1'' = 0, \quad n > 0 \tag{7a}$$

$$\Psi_1'' + \varepsilon_1 \Phi_1 \Psi_1' = 0, \quad \eta > 0 \tag{7b}$$

$$\Phi_2''' + 2\varepsilon_2^{-1} \Phi_2 \Phi_2'' = 0, \quad \eta > 0 \tag{7c}$$

$$\Psi_2'' + 2\varepsilon_2 \Phi_2 \Psi_2' = 0, \quad \eta > 0 \tag{7d}$$

$$\Phi_1 = \theta \Psi_1', \quad \eta = 0 \tag{7e}$$

$$\Phi_1' = 2\theta_1 (\varepsilon_2/\varepsilon_1) \Phi_2', \quad \eta = 0 \tag{7f}$$

$$\Psi_1 = 1 - \Psi_2, \quad \eta = 0 \tag{7g}$$

$$\Phi_2 = 0, \quad \eta = 0 \tag{7h}$$

$$\Phi_2'' = -0.5 \theta_2 (\varepsilon_1/\varepsilon_2)^2 \Phi_1'', \quad \eta = 0 \tag{7i}$$

$$\Psi_2' = (\mathrm{X}/\varepsilon_0) \Psi_1', \quad \eta = 0 \tag{7j}$$

Table 1

	$X/\varepsilon_0 = 1$ $\psi'_2(0) = \psi'_1(0)$		$X/\varepsilon_0 = 0$ $\psi'_2(0) = 0$	
θ	$-\psi'_1(0)$	$\phi_1(0)$	$-\psi'_1(0)$	$\phi_1(0)$
0	0.4510	0	0.7299	0
−0.1	0.4603	0.04503	0.7852	0.07852
0.1	0.4366	−0.04366	0.6822	−0.06822
−0.2	0.4759	0.09517	0.8514	0.1703
0.2	0.4264	−0.08527	0.6414	−0.1283
−0.3	0.4870	0.1460	0.9324	0.2797
0.3	0.4149	−0.1245	0.6054	−0.1816

$\varepsilon_1 = 1$, $\varepsilon_2 = 20$, $\theta_1 = 0.1$, $\theta_2 = 0.152$, $\phi_2(0) = 0$.

Table 2

	$X/\varepsilon_0 = 1$			$X/\varepsilon_0 = 0$		
θ	$\phi'_1(0)$	$\phi''_1(0)$	$\psi_1(0)$	$\phi'_1(0)$	$\phi''_1(0)$	$\psi_1(0)$
0	0.2161	1.304	0.6172	0.2138	1.304	0.9994
−0.1	0.2168	1.361	0.6032	0.2152	1.402	0.9993
0.1	0.2153	1.250	0.6259	0.2129	1.220	0.9994
−0.2	0.2176	1.423	0.5961	0.2166	1.520	0.9993
0.2	0.2147	1.199	0.6348	0.2118	1.148	0.9994
−0.3	0.2190	1.450	0.5831	0.2185	1.662	0.9992
0.3	0.2141	1.153	0.6428	0.2107	1.084	0.9995

$\varepsilon_1 = 1$, $\varepsilon_2 = 20$, $\theta_1 = 0.1$, $\theta_2 = 0.152$, $\phi'_2(0) = 0.25\phi'_1(0)$, $\phi''_2(0) = 1.9\times 10^{-4}\phi''_1(0)$, $\psi_2(0) = 1-\psi_1(0)$.

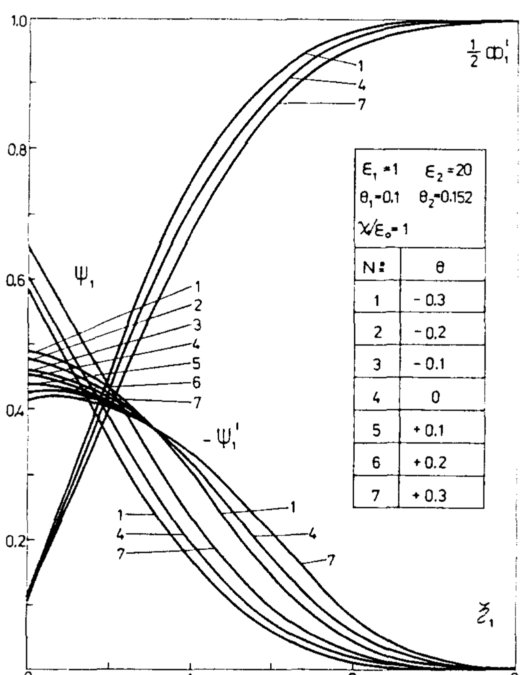

FIG. 1.

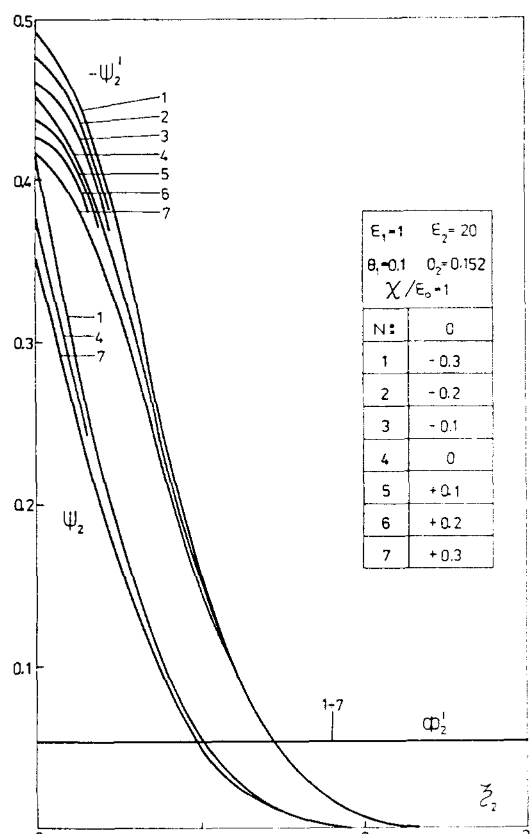

FIG. 2.

$$\Phi_1' = 2\varepsilon_1^{-1}, \quad \eta \to \infty \tag{7k}$$

$$\Psi_1 = 0, \quad \eta \to \infty \tag{7l}$$

$$\Phi_2' = \varepsilon_2^{-1}, \quad \eta \to \infty \tag{7m}$$

$$\Psi_2 = 0, \quad \eta \to \infty \tag{7n}$$

where

$$\theta_1 = u_{20}/u_{10} \tag{8a}$$

$$\theta_2 = (\mu_1/\mu_2)(\nu_1/\nu_2)^{-0.5}(u_{10}/u_{20})^{1.5} \tag{8b}$$

$$\theta = M(\mathrm{X}c_{20} - c_{10})/(\varepsilon_1 \rho_{10}^*) \tag{8c}$$

$$\varepsilon_0 = (\rho_1^*)^{-1}\rho_{10}^*(u_{20}/u_{10})^{0.5}(D_2/D_1)^{0.5}. \tag{8d}$$

In terms of the new similarity variables the Sherwood numbers have the form

$$Sh_1 = k_1 L/D_1 = -(\rho_1^*/\rho_{10}^*)Pe_1^{0.5}\,\Psi_1'(0) \tag{9a}$$

$$Sh_2 = k_2 L/D_2 = -Pe_2^{0.5}\,\Psi_2'(0) \tag{9b}$$

where

$$Pe_1 = u_{10}L/D_1 \tag{9c}$$

$$Pe_2 = u_{20}L/D_2. \tag{9d}$$

The non-linear effects due to the intensive mass transfer influence the kinetics of mass transfer (the Sherwood numbers, respectively) through

$$\rho_1^*/\rho_{10}^* \tag{10a}$$

and the non-dimensional local diffusive fluxes

$$\Psi_k'(0), \quad k = 1, 2. \tag{10b}$$

These quantities depend on θ, while the change of the direction of mass transfer is expressible through the change of the sign of θ. The latter means that this is the only way for the direction of mass transfer to influence the rate of mass transfer.

The solution of the boundary value problem, equations (7), was carried out by the method [6], which is an improved version of the continuation shooting method [7, 8]. Some illustrative results are reported in Tables 1 and 2 and Figs. 1–3.

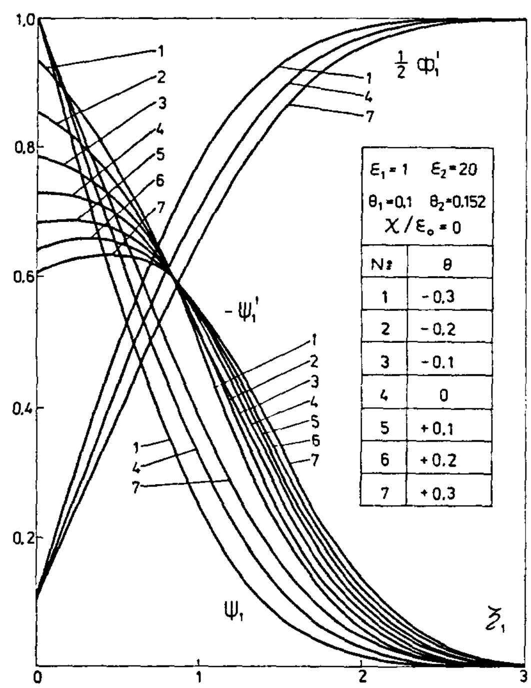

FIG. 3.

3. CONCLUSIONS

These results show that if there is intensive mass transfer in gas–liquid systems, the change of the direction of mass transfer influences significantly the rate of mass transfer in the gas phase when the diffusive resistance in the liquid phase is negligible. Otherwise these non-linear effects are not observed. If the diffusive resistances in both phases are of comparable magnitude, the non-linear effects are present both in the gas and in the liquid, but in the liquid they are a consequence of the diffusion in the gas only, because the hydrodynamics in the gas influences weakly the mass transfer in the liquid.

In all cases the intensive mass transfer from the gas to the liquid results in higher mass transfer rates compared to the data from the linear theory. The intensive mass transfer from the liquid to the gas has the opposite effect.

Acknowledgement—This project was completed with the financial support of the Ministry of Culture, Science and Education under Contract No. 154.

REFERENCES

1. Chr. B. Boyadjiev and N. L. Vulchanov, Effect of the direction of the interphase mass transfer on the rate of mass transfer, *C. R. Bulg. Acad. Sci.* **40**(11), 35–38 (1987).
2. Chr. B. Boyadjiev and N. L. Vulchanov, Influence of the interphase mass transfer on the rate of mass transfer—1. The system 'solid–liquid (gas)', *Int. J. Heat Mass Transfer* **33**, 2039–2044 (1990).
3. Chr. B. Boyadjiev and N. L. Vulchanov, Non-linear mass transfer in boundary layers—1. Asymptotic theory, *Int. J. Heat Mass Transfer* **31**, 795–800 (1988).
4. N. L. Vulchanov and Chr. B. Boyadjiev, Non-linear mass transfer in boundary layers—2. Numerical investigation, *Int. J. Heat Mass Transfer* **31**, 801–805 (1988).
5. Chr. B. Boyadjiev and V. N. Beshkov, *Mass Transfer in Liquid Film Flows*. Publishing House of the Bulgarian Academy of Sciences, Sofia (1984).
6. N. L. Vulchanov and M. M. Konstantinov, An improved continuation procedure for numerical integration of two-point boundary value problems for ordinary differential equations on semi-infinite intervals. EQUADIF 6, Brno, CSSR (1985).
7. J. A. Adams and D. F. Rodgers, *Computer Aided Heat Transfer Analysis*. McGraw-Hill, New York (1973).
8. P. R. Nachtscheim and P. Schwiegert, Satisfaction of asymptotic boundary conditions of systems of nonlinear equations, NASA TN D-3004 (October 1964).

INFLUENCE DU TRANSFERT DE MASSE A L'INTERPHASE SUR LE FLUX DE MASSE—2. LE SYSTEME "GAZ-LIQUIDE"

Résumé—On rapporte des résultats numériques sur l'influence de la direction du transfert de masse à l'interphase sur le flux de masse transféré dans les systèmes "gaz-liquide". On montre que cette influence est significative quand le résistance diffusive de la phase liquide est négligeable. Quand les résistances diffusives dans les deux phases sont du même ordre de grandeur, l'effet est faible. Dans tous les cas le transfert de masse intense du gaz vers le liquide résulte d'un accroissement par rapport au cas de la théorie linéaire. Le changement de direction du transfert de masse a l'effet inverse.

EINFLUSS DES STOFFTRANSPORTS AN DER PHASENGRENZE AUF DIE GESCHWINDIGKEIT DER STOFFÜBERTRAGUNG—2. DAS SYSTEM GAS-FLÜSSIGKEIT

Zusammenfassung—Es werden die Ergebnisse einer numerischen Untersuchung über den Einfluß der Richtung des Stofftransports an der Phasengrenzfläche auf die Geschwindigkeit der Stoffübertragung in einem Gas-Flüssigkeits-System vorgestellt. Es zeigt sich, daß dieser Einfluß dann bedeutsam wird, wenn der Diffusionswiderstand der flüssigen Phase vernachlässigbar ist. Wenn dagegen der Diffusionswiderstand in beiden Phasen von gleicher Größenordnung ist, wird der Einfluß schwach. In allen Fällen führt der intensive Stofftransport vom Gas zur Flüssigkeit dazu, daß die Geschwindigkeit der Stoffübertragung gegenüber der linearen Theorie zunimmt. Eine Umkehr der Richtung des Stofftransports hat den umgekehrten Effekt.

ВЛИЯНИЕ НАПРАВЛЕНИЯ МАССОПЕРЕНОСА НА МЕЖФАЗНОЙ ГРАНИЦЕ НА ЕГО СКОРОСТЬ—2. СИСТЕМА "ГАЗ-ЖИДКОСТЬ"

Аннотация—Приведены численные результаты исследования влияния направления массопереноса на его скорость в системах "газ-жидкость". Пркaзано, что это влияние является существенным при пренебрежимом диффузионном сопротивлении жидкой фазы. В случае, когда диффузионные сопротивления обеих фаз имеют одинаковый порядок величины, указанный эффект незначителен. Во всех случаях интенсивный массоперенос от газа к жидкости приводит к увеличению его скорости по сравнению с результатами линейной теории. Изменение направления массопереноса приводит к обратному эффекту.